Simple Harmonic Motion and Waves

Oscillation
A body is said to be in oscillatory motion when it performs To and fro motion about its mean position. The To and fro motion of a body about its mean position is called oscillation or vibration.

Periodic motion
A type of motion in which a body repeats its motion after regular intervals of time is called periodic motion. For examples
(i) The second’s hand of a clock repeats its motion in every one minute.
(ii) Earth completes its revolution around its axis in 24 hours.
(iii) Motion of the simple pendulum around its mean position.

Q.1 What is Simple Harmonic Motion? Describe its characteristics features.
(Ans) Simple Harmonic Motion
A type of vibratory motion in which acceleration is directly proportional to the displacement from the equilibrium position and is always directed towards the equilibrium position. Mathematically
\[ \alpha \propto -x \]

Characteristics of Simple Harmonic Motion are as under;
   i. A body always executes To and fro motion around a fix point in SHM.
   ii. SHM is always in a straight line. The maximum distance at either side from the mean position is called amplitude of the SHM. OA and OB is called amplitude in figure.
   iii. A restoring force is always acted on the Simple Harmonic Oscillator (SHO).
   iv. The velocity of SHO is maximum at mean position and minimum (zero) at extreme positions.
   v. The restoring force and acceleration of SHO is maximum at extreme positions and minimum (zero) at mean position.

Q.2 Prove that motion of the mass attached to the spring is SHM.
(Ans) Motion of mass attached to a spring
Consider a spring whose one end is attached to a firm support and a mass "m" is connected to the other end of the spring is placed on a frictionless horizontal surface. Initially mass "m" is at rest in its mean position "O" as shown in figure.
If we apply an external force “F”, it produce an extension “x” in the spring and the mass “m” moves from its equilibrium position “O” to extreme position “A” as shown in figure.

Now according to Hooks law the external force “F” acting on the spring is directly proportional to the extension “x”. Mathematically

\[ F_{ext} \propto x \]

\[ F_{ext} = Kx \]

Where k is constant of proportionality which is called the spring constant.

After removing the force “F” the mass “m” moves towards its mean position because of the restoring force. This restoring force is equal to the external force but opposite in direction. Therefore

\[ F_{res} = -Kx \quad \text{-------- (1)} \]

However mass “m” does not stop at mean position due to inertia and move to extreme position “B” as shown in figure.

As a result the body starts Back And Forth motion around its mean position “O” and extreme positions “A” and “B”.

Now according to Newton’s 2nd law of motion

\[ F = ma \quad \text{-------- (2)} \]

Comparing equation (1) and (2) we get

\[ ma = -Kx \]

\[ a = -\frac{K}{m} x \]

\[ a = -\text{cons} \tan t(x) \]

\[ a \propto -x \]

Therefore motion of the mass attached to a spring is SHM.

The time period of mass attached to the spring is find by the formula

\[ T = 2\pi \sqrt{\frac{m}{K}} \]

The frequency of mass attached to the spring is find by the formula

\[ f = \frac{1}{2\pi} \sqrt{\frac{K}{m}} \]
Some important terms used in connection with SHM.

i) **Vibration**
   It can be defined as “one complete round trip of the body around the mean position “O” is called Vibration or oscillation”.

ii) **Time period**
    The time required to complete one vibration or oscillation is called Time period. It is measured in second. It is denoted by “T”. Mathematically
    \[
    T = \frac{t}{n}
    \]
    Where “t” is the time taken and “n” denotes the number of vibrations.

iii) **Frequency**
    It can be defined as “the number of vibrations in one second is called frequency”. It is denoted by “f”. Mathematically
    \[
    f = \frac{n}{t}
    \]
    It can also be defined as “the reciprocal of time period is called frequency”. It is measured in Hertz, cycle per second or vibration per second. Mathematically
    \[
    f = \frac{1}{T}
    \]

iv) **Amplitude**
    The maximum displacement of the body from either side of the mean position is called Amplitude. It is denoted by “x”.

Q.3 **What is Simple pendulum? Illustrate diagrammatically the forces acting on the bob of a Simple pendulum.**

(Ans) **Simple pendulum**
A Simple pendulum is a small metallic bob which is suspended through a weightless and inextensible string connected with a support as shown in figure.
Explaination
If we displace the bob about its mean position “O” to extreme position “A” and then released. The bob starts oscillation around its mean position “O”. Here two forces weight of the bob and tension in the string acts on the pendulum. At mean position “O” weight of the pendulum and tension in the string balance each other. The weight force can be resolved into two rectangular components “Wₓ” and “Wᵧ” at either extreme position. “Wₓ” component of the weight balance tension in the string and “Wᵧ” is responsible to give the restoring force and produce vibrations in string.

Q.4 What is wave? Describe its types.
(Ans) Wave
That agent which carries disturbance (energy) from one place to another in a medium is called wave.

Wave motion
The transmission of energy in a medium due to the oscillatory motion of the particles of the medium about their mean position is called the wave motion.

Generation and propagation of waves
When a stone is dropped into a pond of water, the circular ripples are produced at a place where the stone touches the water. These ripples spread towards the edges in all directions. If a piece of paper is placed at the surface of water and observe, it will start up and down motion at its own position as the ripples pass through it. When the ripples pass through the paper, it stops up and down motion. It means that the disturbance produce is taken by the wave.
Similarly waves can also be produced in a rope. Take a long rope and attached its one end with a firm support. Hold the other end of the rope in your hand and give an upward jerk, you will observe that a pulse shape wave is formed in the rope.
Types of waves are as under;
(1) Mechanical waves
(2) Electromagnetic waves

1. **Mechanical waves**
   A type of waves which require a medium for their production and propagation are called Mechanical waves.

**Examples**
Sound waves, water waves, spring waves, etc are the examples of Mechanical waves.

2. **Electromagnetic waves**
   A type of waves which does not require a medium for their production and propagation are called Electromagnetic waves.

**Examples**
Radio waves, x-rays, light waves, etc are the examples of Electromagnetic waves.

**Types of Mechanical waves**
There are two types of Mechanical waves.

i. **transverse waves**
   A type of waves in which particles of the medium vibrate perpendicularly to the direction of propagation of waves are called Transverse waves. For examples
   Waves produce in pond of water by dropping a stone.
   Waves produce in a stretched string.

**Explanation**
When we give an upward jerk to a string whose one end is tied with a fixed support a pulse of wave is produced as shown in figure.

![Diagram of transverse waves with crests and troughs](image)

The upward portion of the wave pulse is called crest while the lower portion is called trough.

**Crest**
The part of the transverse waves where the medium of propagation is above the mean position is called Crest of the waves.

**Trough**
The part of the transverse waves where the medium of propagation is below the mean position is called Trough of the waves.
ii. **Longitudinal waves**
A type of waves in which particles of the media vibrate parallel to the direction of propagation of waves are called Longitudinal waves. For examples

Sound waves and Waves produced in a compressed spring.

**Explanation**
When we produce a sound, it compresses the molecules of the air and after this compression the nearer space becomes rarefact. These waves move in the form of compressions and rarefaction in the media as shown in figure.

![Diagram of sound waves](image)

**Q.5 Define quantitative characteristics of the waves.**

(Ans) Quantitative characteristics of waves are;

(i) **Wavelength**
The distance between two consecutive crest or trough is called wavelength. OR
The distance between two consecutive compressions or rarefactions is called wavelength. It is denoted by a Greek letter lambda “λ”. Its SI unit is meter.

(ii) **Amplitude of wave**
The maximum displacement of the particles of the medium from their original position is called Amplitude of the wave.

(iii) **Velocity of the wave**
The displacement travel by the wave in unit time is called velocity of the wave. It is denoted by “V”. Mathematically

\[
V = \frac{S}{t}
\]

**Q.6 Using ripple tank explain the following characteristics of wave, reflection, refraction and diffraction.**

(Ans) **Ripple tank**
A device that is used to demonstrate the basic properties of wave.

**Construction**
It consists of a rectangular glass tray filled with water. A lamp is fixed above the tray for throwing light. An electrical vibrator is also
fixed above the tray for production of waves.

**Working**
For the demonstration of wave properties the lamp is lightened and also the vibrator is started to produce waves in the tray. The light shines through the water. The crest in the water is shown by bright lines and trough in the water is shown dark lines. The image of the wave can be seen through the screen placed below. The properties of wave such as reflection, refraction, interference and diffraction can be demonstrated.

**Properties of Waves**

i. **Reflection of waves**
The bounce back of waves in its own media when strike with a resistance is called reflection of waves.
Reflection of waves can be demonstrated in ripple tank by placing a barrier in front of the propagated wave. The incident and reflected waves can be seen on the viewing screen.

ii. **Refraction of waves**
The slightly bending of wave in a certain way when it is passed from one media to another media is called refraction of light.
Refraction of waves can be demonstrated in ripple tank by placing a plastic sheet in the bottom of the tray. It can be seen that the water waves bend with the edges of the plastic sheet.
iii. **Diffraction of waves**

Diffraction is often demonstrated with water waves in a ripple tank. Generate straight waves in a ripple tank and place two obstacles in a line such a way, that separation between them is comparable to the wavelength of water wave. The waves produce in the ripple tank when passes through the opening, becomes spreading in every direction as circular waves.

---

**SHORT QUESTIONS**

1. The mass attached to a vibrating spring is increased four times. What is the affect on the time period and frequency of oscillation of the mass spring system?

(Ans) The time period of the mass spring system becomes double and the frequency becomes half. Mathematically
\[ T = 2\pi \sqrt{\frac{m}{k}} \]
\[ m = 4m \]
\[ T' = 2\pi \sqrt{\frac{4m}{k}} \]
\[ T' = 2(2\pi \sqrt{\frac{m}{k}}) \]
\[ T' = 2T \]

We know that
\[ f' = \frac{1}{T} \]
\[ f' = \frac{1}{2T} \]
\[ f' = \frac{1}{2} f \]

2. A wire hangs from a dark high tower so that its upper end is not visible. How can the length of the wire be determined?

(Ans) A small metallic bob is attached to the lower end of the wire. The wire becomes just like a pendulum. Now calculate the time period of the pendulum by displacing it from the mean position to the extreme position. After the calculation of the time period the length of the wire can be find out by using the formula;
\[ T = 2\pi \sqrt{\frac{l}{g}} \]
Taking square of both sides we get;
\[ T^2 = 4\pi^2 \frac{l}{g} \]
\[ l = \frac{T^2 g}{4\pi^2} \]
3. Will the period of a vibrating swing increase, decrease or remain constant by addition of more weight?
   (Ans) A vibrating string is just like a simple pendulum. The formula of the time period for the simple pendulum is given as:
   \[ T = 2\pi \sqrt{\frac{l}{g}} \]
   This formula shows that, the time period does not depends on weight. So the time period of the string remain constant.

4. Water waves move from the shallow end of a pool to the deeper end. State the changes (if any) to the wavelength and the speed of the wave?
   (Ans) As we know that the speed of water waves is greater in deeper end and slower in the shallow end. Therefore the speed of the waves increasing if the water waves move from shallow end of a pool to the deeper end. We also know that from the equation;
   \[ V = f\lambda \]
   That the wavelength depends on the speed of the waves. Greater the speed greater will be the wavelength.

5. Define the terms Frequency, Amplitude, Time period and wavelength?
   (Ans) Already done in theory.

6. What is the K-E of a Simple pendulum when the bob is at
   i. Mean position  ii. Extreme position
   i. **Mean position**
      At mean position the Kinetic energy of the bob is maximum because velocity of the bob is maximum.
   ii. **Extreme position**
      At extreme position the kinetic energy of the bob is zero because the velocity of the bob is zero.

7. Prove that \( V = f\lambda \)?
   (Ans) As we know that
   \[ \vec{V} = \frac{\vec{S}}{t} \]
   We also know that a wave travels distance \( \lambda \) in time period \( T \). Therefore, equation (i) becomes
\[ V = \frac{\lambda}{T} \]
\[ V = \lambda \times \frac{1}{T} \]
\[ f = \frac{1}{T} \]
\[ V = f\lambda \]

8. The diagram shows a wave moving into shallower water why the wavelength of the wave is reduced.

(Ans) We know that the speed of wave in deep water is greater than the speed of wave in shallow water. We also know that

\[ V = f\lambda \] \hspace{1cm} (1)

Now when a wave enters into shallower water its speed will be decrease. As a result the wavelength of the wave will be decreases.

9. A dipper moving up and down makes waves in ripple tank. What will happen when the dipper frequency is increased?

(Ans) We know that

\[ V = f\lambda \] \hspace{1cm} (1)

Now if the speed of the wave is constant. Then equation (1) becomes

\[ f = \text{constant}\left(\frac{1}{\lambda}\right) \]

\[ f \propto \frac{1}{\lambda} \]

This equation shows that frequency of the wave is inversely proportional to the wavelength. As the frequency of the dipper in the ripple tank increases so its wavelength will be decreases.
NUMERICAL PROBLEMS

(1) The time period of an electromagnetic wave is $10^{-15}$ sec. What is the frequency in (i) Hertz (ii) Mega Hertz?

Given data

- $T = 10^{-15}$ sec
- $f = ?$ Hertz
- $f = ?$ Mega Hertz

We know that

(i) $f = \frac{1}{T}$

$$f = \frac{1}{10^{-15}}$$

$$f = 10^{15} \text{ Hz}$$

(ii) $f = 10^{15} \text{ Hz}$

$$f = 10^{15} \times 10^{-6} \text{ MHz}$$

$$f = 10^9 \text{ MHz}$$

(2) The length of a simple pendulum is 1m. What will be its time period if it is taken to the moon, where the acceleration due to gravity is one-sixth that on the earth? Also calculate the time period on earth surface. (acceleration due to gravity on the earth is $10\text{m/s}^2$).

Given data

- $l = 1\text{m}$
- $g_{moon} = \frac{10}{6} \text{ m/s}^2 = 1.67 \text{ m/s}^2$
- $g_{earth} = 10 \text{ m/s}^2$
- $T_{moon} = ?$
- $T_{earth} = ?$
We know that time period of Simple pendulum is;

\[ T = 2\pi \sqrt{\frac{l}{g}} \]

\[ T_{\text{moon}} = 2(3.14)\sqrt{\frac{1}{1.67}} \]
\[ T_{\text{moon}} = 6.28\sqrt{0.5988} \]
\[ T_{\text{moon}} = (6.28)(0.773) \]
\[ T_{\text{moon}} = 4.85\text{Sec} \]

\[ T_{\text{earth}} = 2(3.14)\sqrt{\frac{1}{10}} \]
\[ T_{\text{earth}} = 6.28\sqrt{0.1} \]
\[ T_{\text{earth}} = (6.28)(0.32) \]
\[ T_{\text{earth}} = 2\text{Sec} \]

(3) Sound waves travel with a speed of 330m/s. What is the wavelength of sound waves whose frequency is 550Hz?

Given data
\[ V = 330\text{m/s} \]
\[ f = 550\text{Hz} \]
\[ \lambda = ? \]

We know that
\[ V = f\lambda \]
\[ \lambda = \frac{V}{f} \]
\[ \lambda = \frac{330}{550} \]
\[ \lambda = 0.6\text{m} \]

(4) The wavelength of red light is 700nm. If the velocity of red light is \(3\times10^8\text{m/s}\). Calculate (a) frequency \(\lambda\), and (b) the time period.

Given data
\[ \lambda = 700\text{nm} = 700\times10^{-9}\text{m} \]
\[ V = 3\times10^8\text{m/s} \]
\[ f = ? \]
\[ T = ? \]
We know that
\[ V = f\lambda \]
\[ f = \frac{V}{\lambda} \]
\[ f = \frac{3 \times 10^8}{700 \times 10^{-9}} \]
\[ f = 0.0042857 \times 10^8 \times 10^9 \]
\[ f = 4.28 \times 10^{14} \text{ Hz} \]
We also know that
\[ T = \frac{1}{f} \]
\[ T = \frac{1}{4.28 \times 10^{14}} \]
\[ T = 0.2336 \times 10^{-14} \]
\[ T = 2.33 \times 10^{-15} \text{ Sec} \]

(5) A pendulum has a frequency \(0.54\text{ Hz}\), if the length of the pendulum is 85cm, calculate g?

Given data
\[ f = 0.54\text{ Hz} \]
\[ l = 85\text{ cm} = 0.85\text{ m} \]
\[ g = ? \]

We know that
\[ f = \frac{1}{2\pi} \sqrt{\frac{g}{l}} \]

Taking square of both sides
\[ f^2 = \left( \frac{1}{2\pi} \sqrt{\frac{g}{l}} \right)^2 \]

\[ f^2 = \frac{1}{4\pi^2} \times \frac{g}{l} \]

\[ 4\pi^2 f^2 l = g \]

\[ g = 4 \times (3.14)^2 \times (0.54)^2 \times 0.85 \]

\[ g = 4 \times 9.86 \times 0.29 \times 0.85 \]

\[ g = 9.72 \text{ m/s}^2 \]